Derivation of LF when the rainfall is ignored.

The Leaching Fraction (LF) equation used in the calculation of Nitrate -nitrogen loading to the ground water is:

LF = [ET*Cli *10^-6 - Clc] / [ET*Clp *10^-6 - Clc]

Where:
LF= Leaching Fraction.
ET= Seasonal evapotranspiration (kg/ha).
Cli= Chloride concentration in the irrigation water (mg/l)
Clp= Chloride concentration in the percolating water below the crop root zone (mg/l).
Clc= Amount of chloride taken up by the crop (kg/ha).

The equation is derived using the definition of leaching fraction and the mass balance equation for chloride

The Leaching Fraction (LF) is defined as:

LF= Vp/Vi ..... (1)

where:
Vp: Volume of percolating water below the crop root zone (l).
Vi: Volume of irrigation water (l).

The volume of irrigation water is:

Vi= Vp + ET ....(2)

where ET is seasonal evapotranspiration (l)

Substituting eq.2 into eq. 1 results in eq 3.

LF= Vp/(Vp + ET).....(3)

By Mass balance under steady state conditions, the chloride input equals the chloride output

input=output

Cli * Vi = (Clp * Vp) + Clc ..... (4)

Where:
Cli= Chloride concentration in the irrigation water (mg/l).
Vi = Volume of irrigation water (l).
Clp= Chloride concentration in the percolating water below the crop root zone (mg/l).
Vp= Volume of percolating water below the crop root zone (l).
Clc= Amount of chloride taken up by the crop (mg).

Substituting eq. 2 into eq. 4 and solving for Vp results in eq. 5

Cli *(Vp + ET) = (Clp * Vp) + Clc
Cli * Vp + Cli * ET = Clp * Vp + Clc
Cli* ET - Clc = Clp * Vp - Cli * Vp
Cli * ET - Clc = Vp (Clp - Cli)

Vp = [(Cli * ET) -Clc] / [Clp - Cli] ..... (5)

Solving eq. 3 for Vp results in eq. 6

Vp = [LF * ET] / [1-LF] ..... (6)

Substituting eq. 5 into eq. 6 and changing ET and Clc units to kg/ha and then solving for LF results in eq. 7

Vp = (LF * ET) / (1- LF) =[(Cli * ET) - Clc] / [Clp-Cli]
[Clp - Cli][LF * ET] = [1-LF][(Cli * ET) - Clc]
LF*ET*Clp - LF*ET*Cli = ET*Cli - Clc - LF*ET*Cli +LF*Clc
LF(ET*Clp - ET*Cli +Cli*ET - Clc) = Cli*Et - Clc

LF= [ET*Cli - Clc]/ [ET*Clp - Clc] = [ET*Cli *10^-6 - Clc] / [ET*Clp *10^-6 - Clc].....(7)
Where:
LF= Leaching Fraction.
ET= Seasonal evapotranspiration (kg/ha).
Cli= Chloride concentration in the irrigation water (mg/l)
Clp= Chloride concentration in the percolating water below the crop root zone (mg/l).
Clc= Amount of chloride taken up by the crop (kg/ha).

Leaching fraction calculation including rainfall

Vr=volume of rainfall
Clr= chloride of rainfall

Volume of the irrigation plus rainfall is:
Vr+Vi= Vp+ET
Vi=Vp+ET-Vr ..... (8)
Mass Balance of the chloride is:

Cli*Vi +Clr*Vr= Clp*Vp + Clc..... (9)

Subsitute eq 8 into 9 and solve for Vp

Cli(Vp+ET_Vr)+(Clr*Vr=(Clp*Vp)+Clc
Cli*Vp+Cli*ET-Cli*Vr+ Clr* Vr=Clp*Vp +Clc
Cli*Vp-Clp*Vp= Clc-Clr*Vr+Cli*Vr-Cli*Et
Vp=Clc-Clr*Vr+Cli*Vr-Cli*Et/ Cli-Clp.....(10)

Take eq. 6 and substitute equation 10 into eq. 6

Vp= LF*ET/ 1-LF .....copy of eq.(6)

LF*ET/1-LF= Clc-Clr*Vr+Cli*Vr-Cli*Et/ Cli-Clp

:LF*Et(Cli-Clp)= (l-LF)(Clc-Clr*Vr+Cli*Vr-Cli*Et)

(LF*Et*Cli) - (LF*ET*Clp)= Clc-(LF*Clc)-(Clr*Vr)+(LF*Clr*Vr)+(Cli*Vr)-(LF*Cli*Vr)-(Cli*Et)+(LF*Cli*ET)

Simplify remove left and right side Lf*Et*Cli

- (LF*ET*Clp)= Clc-(LF*Clc)-(Clr*Vr)+(LF*Clr*Vr)+(Cli*Vr)-(LF*Cli*Vr)-(Cli*Et)+(LF*Cli*ET)

move terms with Lf on left side
-(LF*ET*Clp)+(LF*Clc)-(LF*Clr*Vr)+(Lf*Cli*Vr)= Clc- (Clr*Vr)+(Cli*Vr)- (Cli*ET)

Lf(-ET*Clp+Clc- (Clr*Vr)+(Cli*Vr))= Clc+Vr(Cli-Clr)-Cli*Et

LF= Clc+Vr(Cli-Clr) - Cli*ET/ (Cli*Vr) - (Clr*Vr) -ET*Clp+Clc

Multiple numerator and denominator by - sign

LF= Cli*ET-Clc- Vr(Cli-Clr)/ Et* Clp - Clc -Vr*(Cli-Clr) ..... eq. 11

LF= Cli*ET10^-6-Clc- Vr(Cli-Clr)10^-6/ Et* Clp10^-6 - Clc -Vr*(Cli-Clr)10^-6 ..... eq. 12

ET and Vr and Clc in units of Kg/ha and Cl in units of (mg/l)