Derivation of Leaching fraction calculation including rainfall

The Leaching Fraction (LF) equation used in the calculation of Nitrate -nitrogen loading to the ground water is:

LF= [Cli*ET10^-6-Clc- Vr(Cli-Clr)10^-6]/ [Et* Clp10^-6 - Clc -Vr*(Cli-Clr)10^-6]

Where:
LF= Leaching Fraction.
ET= Seasonal evapotranspiration (kg/ha).
Cli= Chloride concentration in the irrigation water (mg/l)
Clp= Chloride concentration in the percolating water below the crop root zone (mg/l).
Clc= Amount of chloride taken up by the crop (kg/ha).
Vr=volume of rainfall Kg/ha
Clr= chloride of rainfall(mg/l)

The equation is derived using the definition of leaching fraction and the mass balance equation for chloride

The Leaching Fraction (LF) is defined as:

LF= Vp/Vi + Vr..... (1)

where:
Vp: Volume of percolating water below the crop root zone (l).
Vi: Volume of irrigation water
Vr Volume of rainfall or water supplied from a second source of irrigation water. (i.e. surface water source 1 ground water source 2 both having different chloride content)

Volume of the irrigation plus rainfall is:

Vr+Vi= Vp+ET
Vi=Vp+ET-Vr ..... (2)
where ET is seasonal evapotranspiration (l)

Substituting eq.2 into eq. 1 and solving for VP results in eq 3.

Vp= [LF*ET]/ [1-LF].....(3)

By Mass balance under steady state conditions, the chloride input equals the chloride output input=output

Cli*Vi +Clr*Vr= Clp*Vp + Clc..... (4)

Subsitute eq 2 into 4 and solve for Vp

Cli(Vp+ET_Vr)+(Clr*Vr=(Clp*Vp)+Clc
Cli*Vp+Cli*ET-Cli*Vr+ Clr* Vr=Clp*Vp +Clc
Cli*Vp-Clp*Vp= Clc-Clr*Vr+Cli*Vr-Cli*Et
Vp=Clc-Clr*Vr+Cli*Vr-Cli*Et/ Cli-Clp......(4)

Take eq. 3 and substitute into equation 4

LF*ET/1-LF= Clc-Clr*Vr+Cli*Vr-Cli*Et/ Cli-Clp

:LF*Et(Cli-Clp)= (l-LF)(Clc-Clr*Vr+Cli*Vr-Cli*Et)

(LF*Et*Cli) - (LF*ET*Clp)= Clc-(LF*Clc)-(Clr*Vr)+(LF*Clr*Vr)+(Cli*Vr)-(LF*Cli*Vr)-(Cli*Et)+(LF*Cli*ET)

Simplify remove left and right side Lf*Et*Cli

- (LF*ET*Clp)= Clc-(LF*Clc)-(Clr*Vr)+(LF*Clr*Vr)+(Cli*Vr)-(LF*Cli*Vr)-(Cli*Et)+(LF*Cli*ET)

move terms with Lf on left side
-(LF*ET*Clp)+(LF*Clc)-(LF*Clr*Vr)+(Lf*Cli*Vr)= Clc- (Clr*Vr)+(Cli*Vr)- (Cli*ET)

Lf(-ET*Clp+Clc- (Clr*Vr)+(Cli*Vr))= Clc+Vr(Cli-Clr)-Cli*Et

LF= Clc+Vr(Cli-Clr) - Cli*ET/ (Cli*Vr) - (Clr*Vr) -ET*Clp+Clc

Multiple numerator and denominator by - sign

LF= Cli*ET-Clc- Vr(Cli-Clr)/ Et* Clp - Clc -Vr*(Cli-Clr) ..... eq. 5

LF= [Cli*ET10^-6-Clc- Vr(Cli-Clr)10^-6]/[Et* Clp10^-6 - Clc -Vr*(Cli-Clr)10^-6] .....eq. 6

ET and Vr and Clc in units of Kg/ha and Cli , Clr, and Clp are in units of (mg/l)